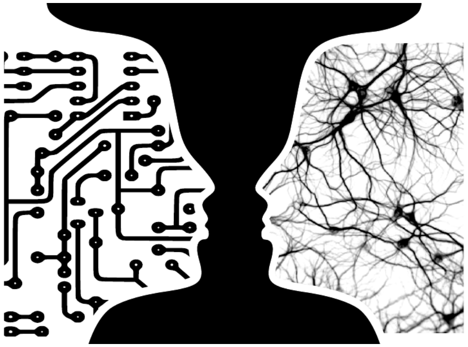
Adaptation de domaine en apprentissage automatique:

Introduction et approche PAC-Bayésienne

Pascal Germain

Groupe de **R**echerche en **A**pprentissage **A**utomatique de l'Université **L**aval



Travail conjoint avec

François Laviolette (Université Laval),

Emilie Morvant (Aix-Marseille Université),

Amaury Habrard (Université Jean-Monnet de Saint-Étienne)

5 avril 2013

Plan de match

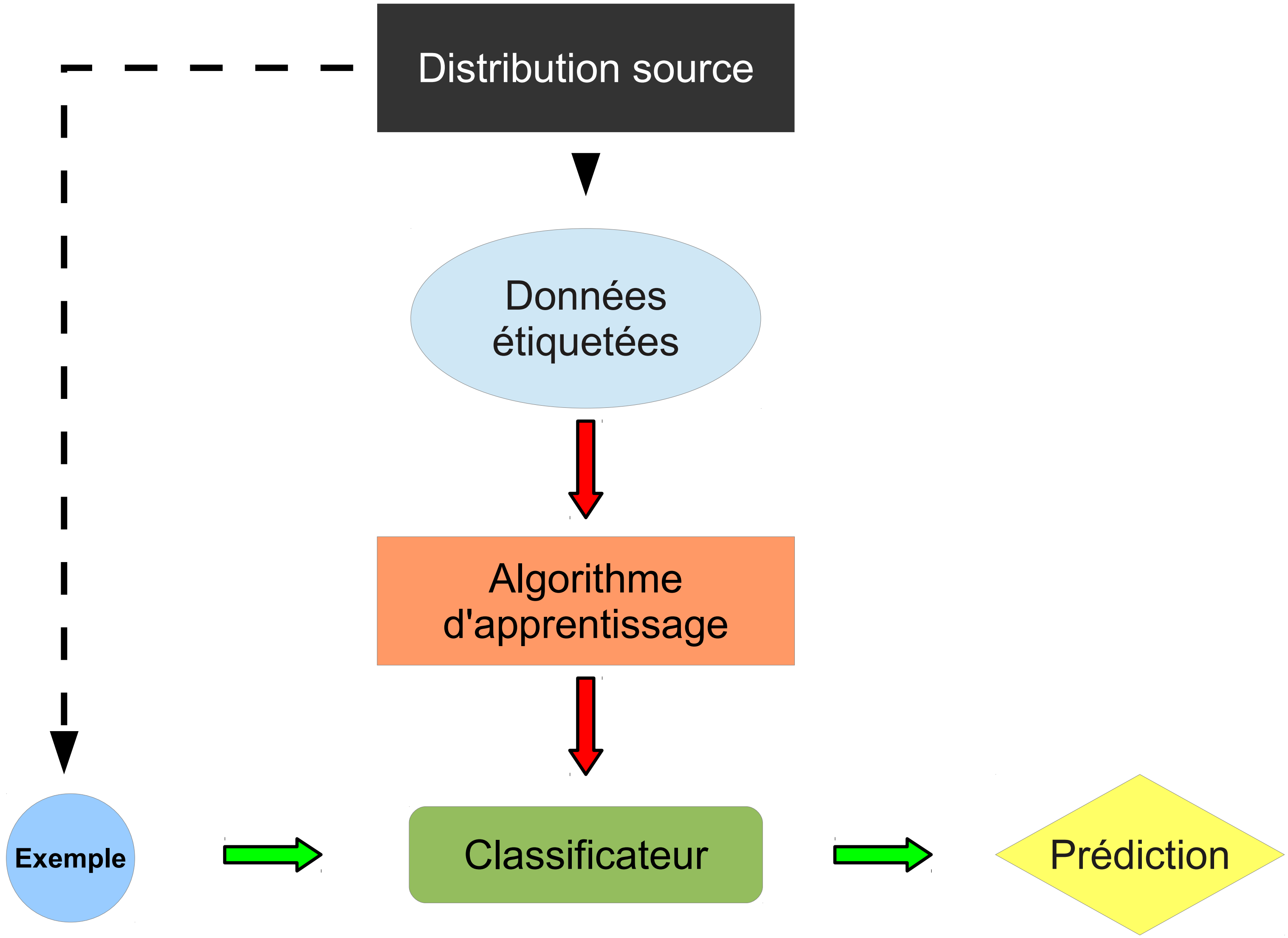
1) Classification classique ***vs*** adaptation de domaine
2) Problème de classification classique
   - Représentation des données
   - Classificateur à noyau
   - Apprentissage PAC-Bayésien
3) Problème d'adaptation de domaine
   - Représentation des données
   - Divergence entre les domaines
   - Apprentissage PAC-Bayésien
4) Résultats empiriques

Classification classique

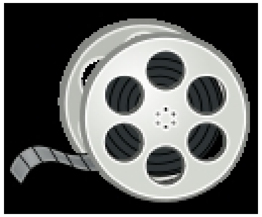
vs

Adaptation de domaine

Problème de classification classique



Exemple



critiques de films

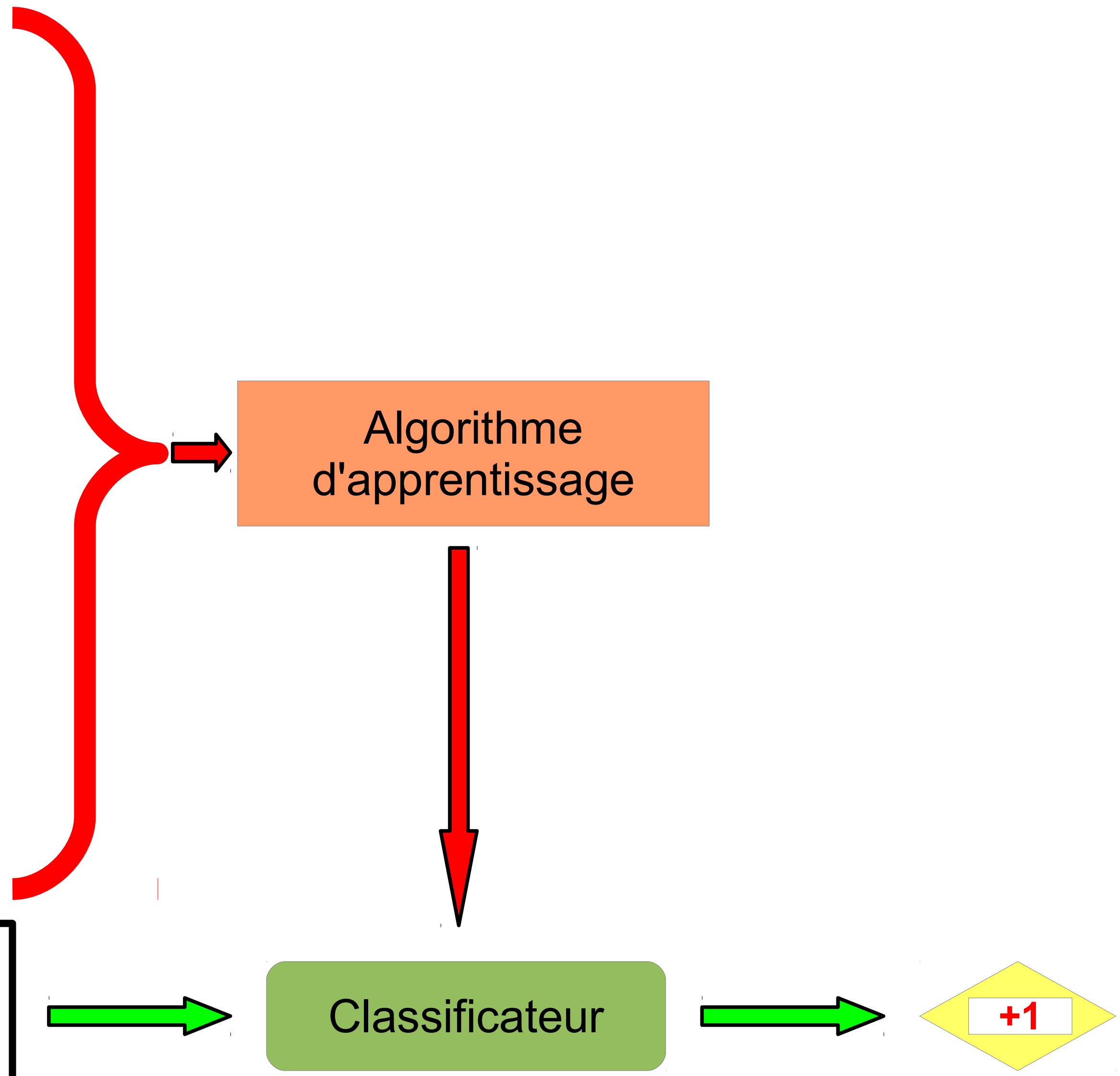
-1 **An insult to Douglas Adams' memory**
I agree entirely with "darkgenius'" comments. This movie is a travesty of the book and the TV series; a cutesy version totally lacking in the wit and satire of the original. Read more
Published 5 months ago by John W Beare

+1 **Don't Panic!**
If you haven't listened to the BBC radio-play, this isn't bad! Purists, no doubt, will dispute my verdict but the fact of the matter is THGTTG (see title) does have Douglas Adams'...
Read more
Published on Mar 13 2011 by Sid Matheson

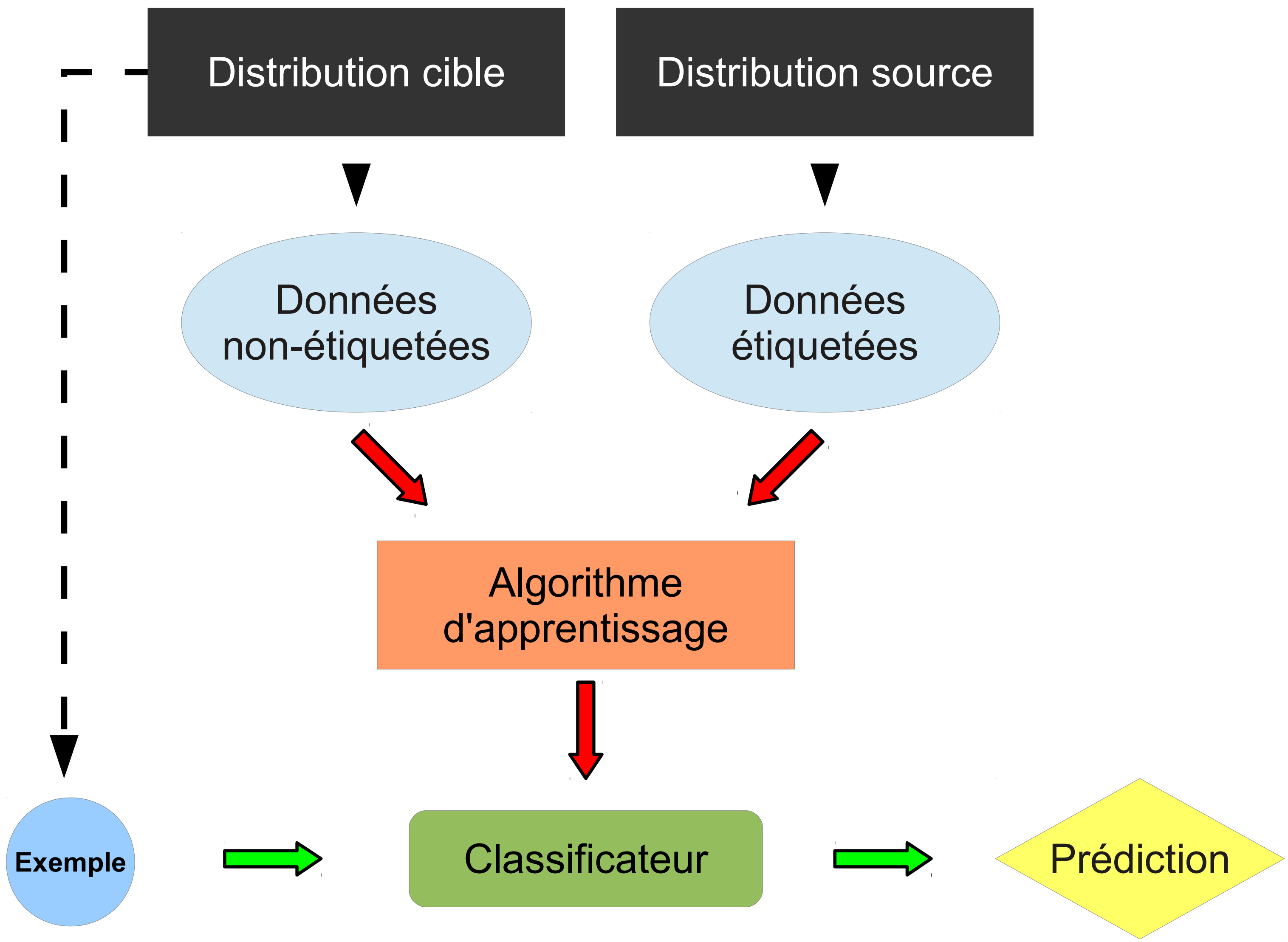
+1 **On Blu-ray, even better**
I've seen this movie on TV and wanted to add it to my collection. I couldn't find it locally so when I saw it on amazon and on Blu-ray, I picked it up. Read more
Published on April 18 2009 by J. W. Little

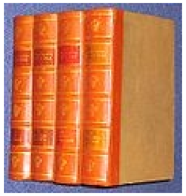
-1 **An insult to Douglas Adams' memory**
The filmmaker's reverence for Adams' legacy? What kind of rubbish statement is that? As a loyal fan of Douglas Adams for more than a quarter of a century, I was appalled and...
Read more
Published on Aug 22 2006 by Daniel Jolley

??? **Mindbending**
I will not recommend this movie for people who haven't read at least two or three of Douglas Adams' books on hitchhiking. Read more
Published on Mar 28 2006 by alper bac

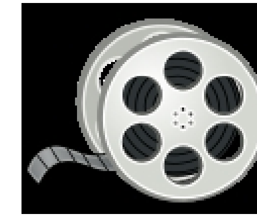


Problème d'adaptation de domaine





Exemple



critiques de livres

??? **The end of the series.**
This book was written to provoke those who wanted Adams to continue the trilogy but I loved it. Aurthor setteled down on a bob fearing planet where he has aquired the prestigous...
Read more
Published on Mar 18 2002 by dan

??? **Mostly Harmless is Underrated**
I think most of the reviews for this book downplay it seriously. While the ending is kind of disappointing, the book overall is wonderful.
Read more
Published on Jan 22 2002 by A Big Adams Fan

??? **Please pretend this book was never written.**
I have long been a fan of the Hitchhikers series as they are comic genius. The book Mostly Harmless, however, should never have come about. It is frustration at its peak. Read more
Published on Jan 14 2002 by Paul Norrod

??? **Kinda like horror movies...**
...in that the last one usually isn't all that appealing. I liked it fine, with some of Adams's wit, but it was a bit disappointing. Read more
Published on Nov 4 2001 by Kristopher Vincent

critiques de films

-1 **An insult to Douglas Adams' memory**
I agree entirely with "darkgenius'" comments. This movie is a travesty of the book and the TV series; a cutesy version totally lacking in the wit and satire of the original. Read more
Published 5 months ago by John W Beare

+1 **Don't Panic!**
If you haven't listened to the BBC radio-play, this isn't bad! Purists, no doubt, will dispute my verdict but the fact of the matter is THGTTG (see title) does have Douglas Adams'...
Read more
Published on Mar 13 2011 by Sid Matheson

+1 **On Blu-ray, even better**
I've seen this movie on TV and wanted to add it to my collection. I couldn't find it locally so when I saw it on amazon and on Blu-ray, I picked it up. Read more
Published on April 18 2009 by J. W. Little

-1 **An insult to Douglas Adams' memory**
The filmmaker's reverence for Adams' legacy? What kind of rubbish statement is that? As a loyal fan of Douglas Adams for more than a quarter of a century, I was appalled and...
Read more
Published on Aug 22 2006 by Daniel Jolley

Algorithme d'apprentissage

??? **A Terrible End to A Great Series**
The ending for this books was so bad that I vowed never to read another Douglas Adams book. Adams was obviously sick and tired of the series and used this book to kill it off with...
Read more
Published on Oct 17 2001 by David A. Lessnau

Classificateur

-1

Problème de classification classique

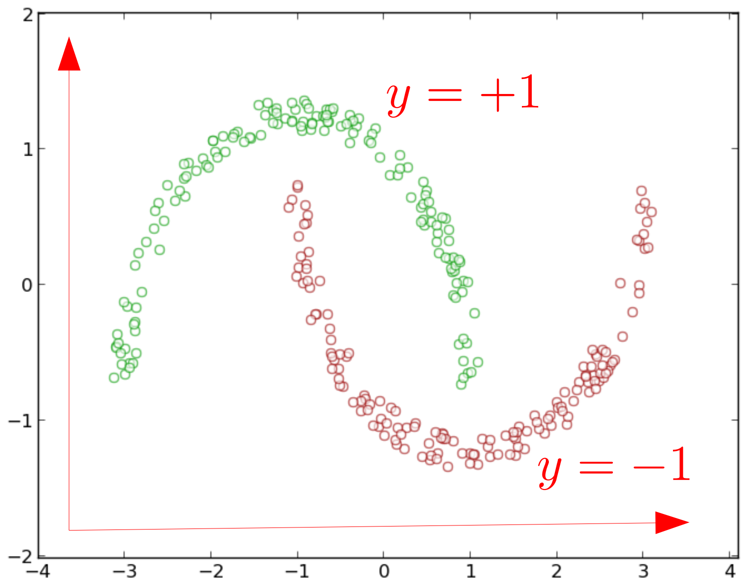
Représentation des données

- Chaque exemple est une paire $(\mathbf{x}, y) \sim D$
  - Description $\mathbf{x} \in \mathbb{R}^d$
  - Étiquette $y \in \{-1, +1\}$

Distribution source

- Ensemble d'entraînement:

$$S = \{(\mathbf{x}_1, y_1), (\mathbf{x}_2, y_2), \ldots, (\mathbf{x}_m, y_m)\} \sim D^m$$



Classificateur à noyau (*kernel*)

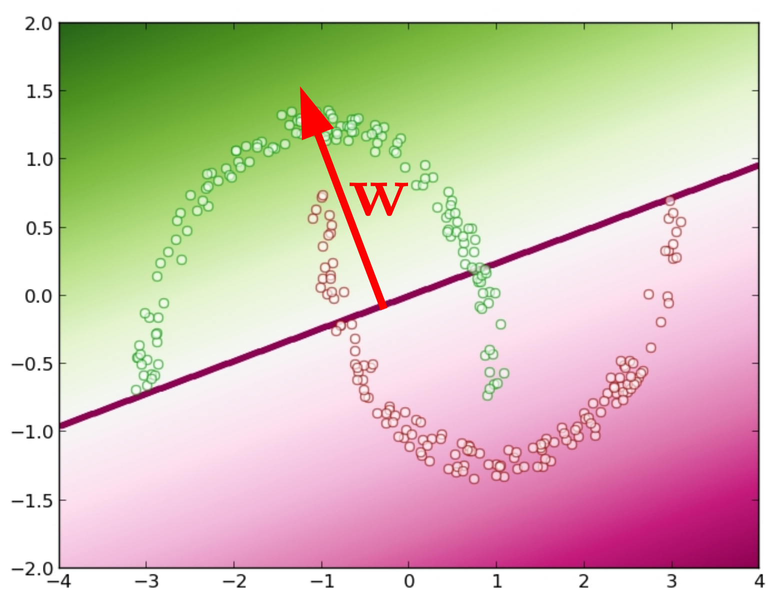
- Classificateur linéaire: exprimé par un vecteur de poids $\mathbf{w} \in \mathbb{R}^d$

- Noyau: Exprime un produit scalaire $k(\mathbf{x}, \mathbf{x}') \rightarrow \mathbb{R}$

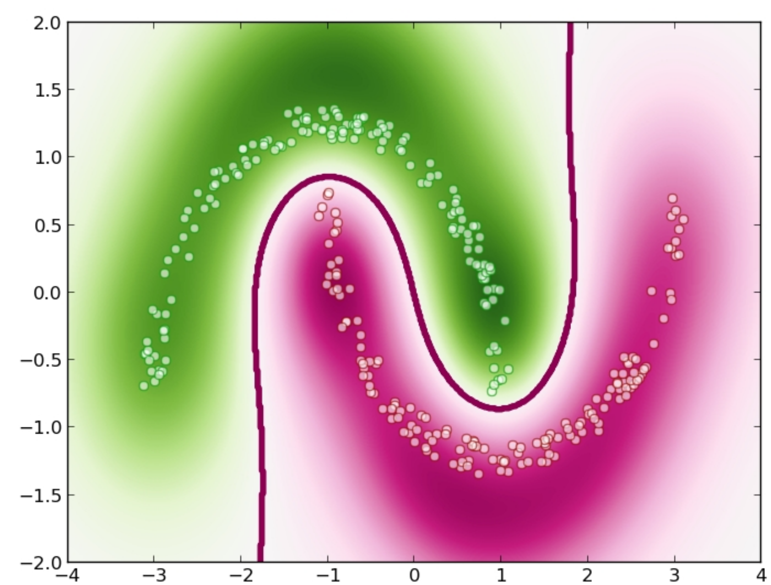
- Classificateur à noyau: Classificateur linéaire dans un espace augmenté exprimé comme une combinaison linéaire des exemples $\boldsymbol{\alpha} \in \mathbb{R}^m$

$$\mathbf{w} \cdot \mathbf{x} = \sum_{i=1}^{m} \alpha_i k(\mathbf{x}_i, \mathbf{x})$$

$$h_{\mathbf{w}}(\mathbf{x}) = \mathrm{sgn}\left(\mathbf{w} \cdot \mathbf{x}\right)$$



<u>Noyau Gaussien :</u> $k(\mathbf{x}, \mathbf{x}') = e^{\frac{\|\mathbf{x}-\mathbf{x}'\|^2}{2\sigma^2}}$



Apprentissage PAC-Bayésien

Théorème PAC-Bayes (spécialisé aux classificateurs linéaires)

(Langford et Shawe-Taylor, 2002)

(Catoni, 2007)

- Pour tout domaine D
- Pour toute *confiance* $\delta \in (0,1]$
- Pour tout paramètre $c \in (0,\infty)$

$$\Pr_{S\sim D^m}\left(\forall \mathbf{w}\in\mathbb{R}^d \;:\; R_D(G_\mathbf{w}) \le \frac{1}{1-e^{-c}}\left[c\cdot R_S(G_\mathbf{w}) + \frac{\frac{1}{2}\|\mathbf{w}\|^2 + \ln\frac{1}{\delta}}{m}\right]\right) \;\ge\; 1-\delta\,.$$

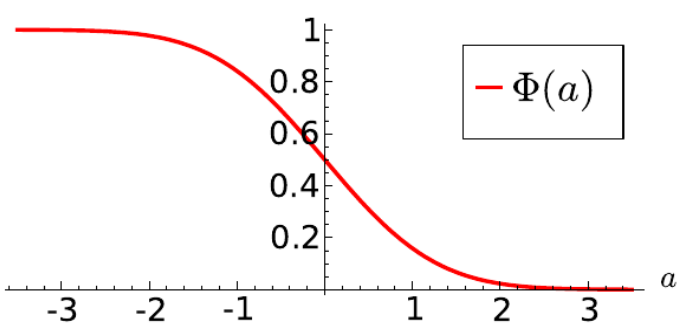
Risque

$$R_D(G_\mathbf{w}) = \mathop{\mathbf{E}}_{(\mathbf{x},y)\sim D} \Phi\left(y\frac{\mathbf{w}\cdot\mathbf{x}}{\|\mathbf{x}\|}\right)$$

Risque empirique

$$R_S(G_\mathbf{w}) = \sum_{i=1}^{m} \Phi\left(y_i\frac{\mathbf{w}\cdot\mathbf{x}_i}{\|\mathbf{x}_i\|}\right)$$

Régularisateur



Apprentissage PAC-Bayésien

Théorème PAC-Bayes (spécialisé aux classificateurs linéaires)

(Langford et Shawe-Taylor, 2002)

(Catoni, 2007)

- Pour tout domaine D
- Pour toute *confiance* $\delta \in (0,1]$
- Pour tout paramètre $c \in (0,\infty)$

$$\Pr_{S\sim D^m}\left(\forall \mathbf{w}\in\mathbb{R}^d \;:\; \underbrace{R_D(G_\mathbf{w})}_{} \leq \frac{1}{1-e^{-c}}\left[c\cdot \underbrace{R_S(G_\mathbf{w})}_{} + \underbrace{\frac{\frac{1}{2}\|\mathbf{w}\|^2+\ln\frac{1}{\delta}}{m}}_{}\right]\right) \;\geq\; 1-\delta\,.$$

Risque

$$R_D(G_\mathbf{w}) = \mathop{\mathbf{E}}_{(\mathbf{x},y)\sim D} \Phi\left(y\frac{\mathbf{w}\cdot\mathbf{x}}{\|\mathbf{x}\|}\right)$$

Risque empirique

$$R_S(G_\mathbf{w}) = \sum_{i=1}^{m} \Phi\left(y_i\frac{\mathbf{w}\cdot\mathbf{x}_i}{\|\mathbf{x}_i\|}\right)$$

Régularisateur

Algorithme d'apprentissage

(Germain et al., 2009)

Étant donné un ensemble S de m exemples et un paramètre C :

Trouver le $\mathbf{w}$ qui minimise : $$C\cdot m\, \overbrace{R_S(G_\mathbf{w})}^{} + \overbrace{\frac{1}{2}\|\mathbf{w}\|^2}^{}$$

Problème d'adaptation de domaine

Représentation des données

- Exemples sources $(\mathbf{x}^s, y^s)$ ◄ Distribution source
  - Description $\mathbf{x}^s \in \mathbb{R}^d$
  - Étiquette $y^s \in \{-1, +1\}$
- Exemples cibles $(\mathbf{x}^t, y^t) \sim D_T$ ◄ Distribution cible
  - Description $\mathbf{x}^t \in \mathbb{R}^d$
  - Étiquette $y^t \in \{-1, +1\}$

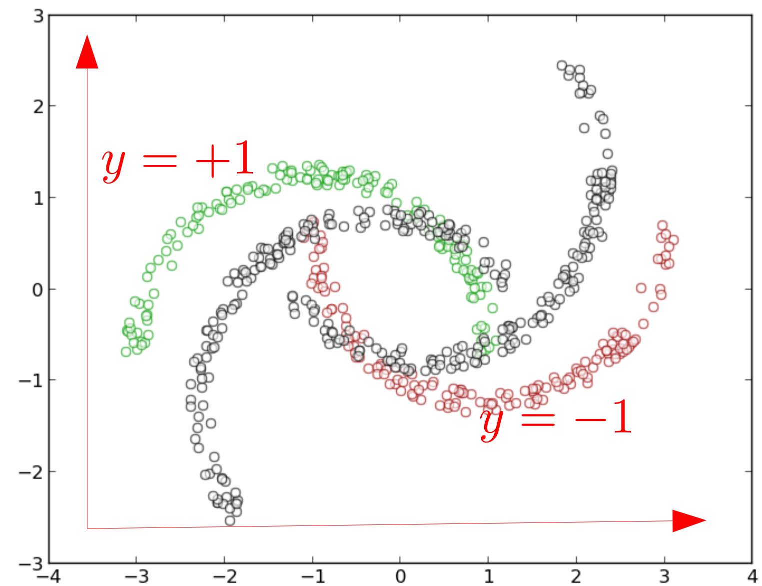
Représentation des données

- Exemples sources $(\mathbf{x}^s, y^s)$
  - Description $\mathbf{x}^s \in \mathbb{R}^d$
  - Étiquette $y^s \in \{-1, +1\}$

Distribution source

- Exemples cibles $(\mathbf{x}^t, \cancel{y^t}) \sim D_{T'}$
  - Description $\mathbf{x}^t \in \mathbb{R}^d$
  - ~~Étiquette~~ $\cancel{y^t \in \{-1, +1\}}$

Distribution cible



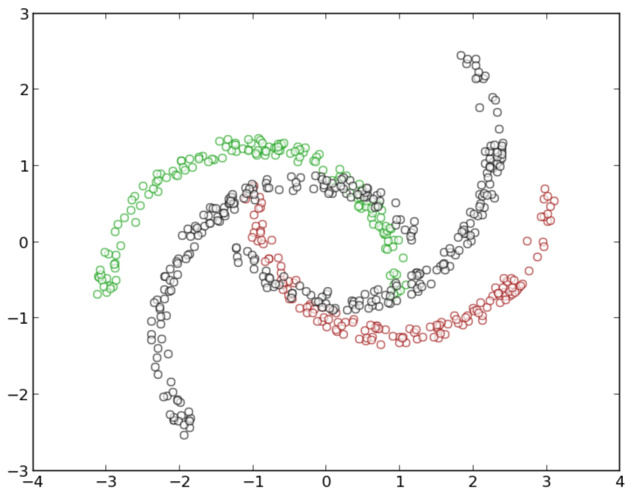
- Ensembles d'entraînement

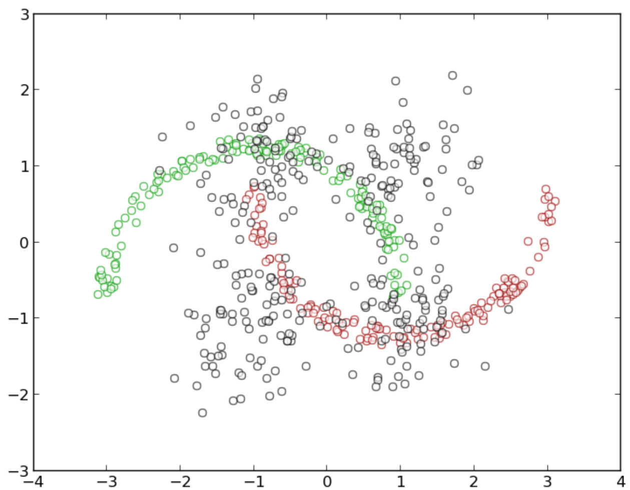
$$S = \{(\mathbf{x}_1^s, y_1^s), \ldots, (\mathbf{x}_m^s, y_m^s)\} \sim D_S^m$$

$$T = \{\mathbf{x}_1^t, \ldots, \mathbf{x}_m^t\} \sim D_{T'}^m$$

Quand l'adaptation est-elle possible?

- **Question :** Dans quelle(s) situation(s) est-il possible de s'adapter au domaine cible?
- **Réponse (partielle) :** Lorsque les domaines source et cible sont « semblables ».
- On a donc besoin d'une mesure permettant de quantifier la **« distance »** entre les distributions.





Divergence entre les distributions

Théorème *(Ben-David et al., 2010)*

Soit D_S une distribution source et D_T une distribution cible.

Soit $\mathcal{H}$ une famille de classificateurs. Pour tout $h \in \mathcal{H}$, on a

$$\underbrace{R_{D_T}(h)}_{\text{Risque cible}} \leq \underbrace{R_{D_S}(h)}_{\text{Risque source}} + \underbrace{\frac{1}{2} d_{\mathcal{H}\Delta\mathcal{H}}(D_{S'}, D_{T'})}_{\text{Divergence}} + \underbrace{\min_{h^* \in \mathcal{H}} \left[R_{D_T}(h^*) + R_{D_S}(h^*) \right]}_{\text{Meilleur risque conjoint}}$$

Risque cible

Risque source

Meilleur risque conjoint

Divergence

$$\frac{1}{2} d_{\mathcal{H}\Delta\mathcal{H}}(D_{S'}, D_{T'}) = \sup_{h,h' \in \mathcal{H}} \left| \mathop{\mathbf{E}}_{\mathbf{x}^s \sim D_{S'}} \mathbf{I}[h(\mathbf{x}^s) \neq h'(\mathbf{x}^s)] - \mathop{\mathbf{E}}_{\mathbf{x}^t \sim D_{T'}} \mathbf{I}[h(\mathbf{x}^t) \neq h'(\mathbf{x}^t)] \right|$$

Divergence entre les distributions

Nouveau théorème (spécialisé aux classificateurs linéaires)

Soit D_S une distribution source et D_T une distribution cible.

Pour tout $\mathbf{w} \in \mathbb{R}^d$, représentant un classificateur linéaire, on a

$$R_{D_T}(G_{\mathbf{w}}) \leq \boxed{R_{D_S}(G_{\mathbf{w}}) + \mathrm{dis}_{\mathbf{w}}(D_{S'}, D_{T'})} + R_{D_T}(G^*) + \lambda^*_{\mathbf{w}}$$

Risque cible

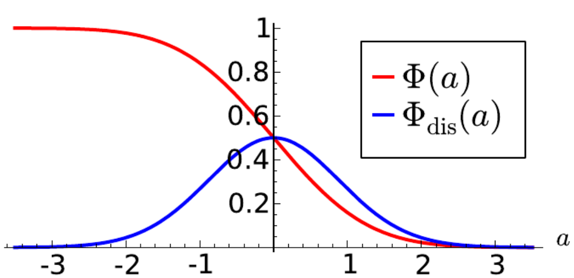
$$R_{D_T}(G_{\mathbf{w}}) = \mathop{\mathbf{E}}_{(\mathbf{x}^t, y^t)\sim D_T} \Phi\left(y^t \frac{\mathbf{w}\cdot\mathbf{x}^t}{\|\mathbf{x}^t\|}\right)$$

Risque source

$$R_{S_T}(G_{\mathbf{w}}) = \mathop{\mathbf{E}}_{(\mathbf{x}^s, y^s)\sim D_S} \Phi\left(y^s \frac{\mathbf{w}\cdot\mathbf{x}^s}{\|\mathbf{x}^s\|}\right)$$

Meilleur risque cible

Mesure de capacité d'adaptation



Divergence

$$\mathrm{dis}_{\mathbf{w}}(D_{S'}, D_{T'}) = \left| \mathop{\mathbf{E}}_{\mathbf{x}^s\sim D_{S'}} \Phi_{\mathrm{dis}}\left(\frac{\mathbf{w}\cdot\mathbf{x}^s}{\|\mathbf{x}^s\|}\right) - \mathop{\mathbf{E}}_{\mathbf{x}^t\sim D_{T'}} \Phi_{\mathrm{dis}}\left(\frac{\mathbf{w}\cdot\mathbf{x}^t}{\|\mathbf{x}^t\|}\right) \right|$$

Apprentissage PAC-Bayésien

Nouveau théorème PAC-Bayes (spécialisé aux classificateurs linéaires)

- Pour tout domaines D_S et D_T
- Pour toute *confiance* $\delta \in (0,1]$
- Pour tout paramètres $c \in (0,\infty)$ et $\alpha \in (0,\infty)$

$$\Pr_{\substack{(S\times T)\sim\\(D_S\times D_T)^m}}\left(\begin{array}{l}\forall \mathbf{w}\in\mathbb{R}^d \ : \boxed{R_{D_T}(G_\mathbf{w}) + \mathrm{dis}(D_{S'},D_{T'})}\\ \qquad \le c'\cdot R_S(G_\mathbf{w}) + \alpha'\cdot \mathrm{dis}(S,T) + \left(\frac{c'}{c}+\frac{2\alpha'}{\alpha}\right)\frac{\frac{1}{2}\|\mathbf{w}\|^2 + \ln\frac{3}{\delta}}{m}\end{array}\right) \ge 1-\delta\,.$$

avec $c' = \frac{c}{1-e^{-c}}$ et $\alpha' = \frac{2\alpha}{1-e^{-\alpha}}$

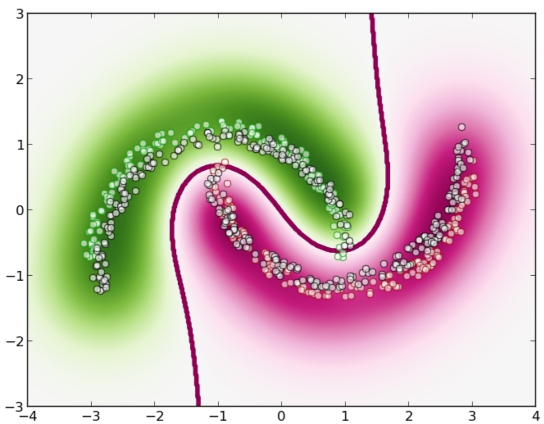
Algorithme d'apprentissage

Étant donné des ensembles S et T de m exemples et deux paramètres C et A, **trouver le $\mathbf{w}$ qui minimise :**

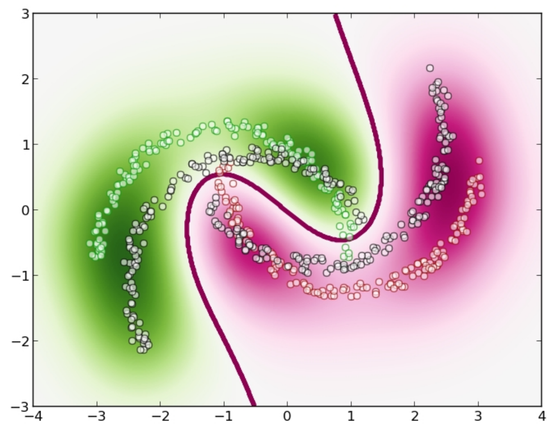
$$C\, m\, \underbrace{R_S(G_\mathbf{w})}_{\text{Risque source empirique}} + A\, m\, \underbrace{\mathrm{dis}_\mathbf{w}(S,T)}_{\text{Divergence empirique}} + \underbrace{\tfrac{1}{2}\|\mathbf{w}\|^2}_{\text{Régularisateur}}$$

Résultats empiriques

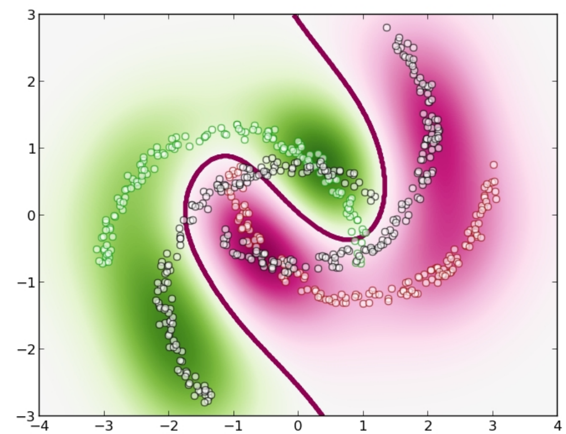
Problème jouet



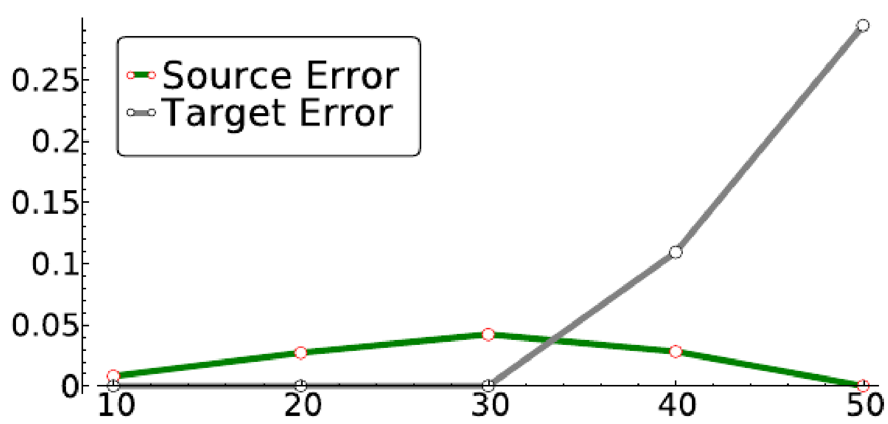
10 degrés

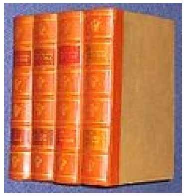


30 degrés

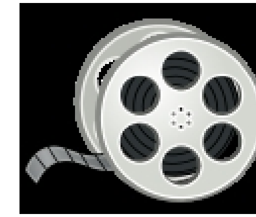


50 degrés





Amazon reviews



critiques de livres

??? **The end of the series.**
This book was written to provoke those who wanted Adams to continue the trilogy but I loved it. Aurthor setteled down on a bob fearing planet where he has aquired the prestigous...
Read more
Published on Mar 18 2002 by dan

??? **Mostly Harmless is Underrated**
I think most of the reviews for this book downplay it seriously. While the ending is kind of disappointing, the book overall is wonderful.
Read more
Published on Jan 22 2002 by A Big Adams Fan

??? **Please pretend this book was never written.**
I have long been a fan of the Hitchhikers series as they are comic genius. The book Mostly Harmless, however, should never have come about. It is frustration at its peak. Read more
Published on Jan 14 2002 by Paul Norrod

??? **Kinda like horror movies...**
...in that the last one usually isn't all that appealing. I liked it fine, with some of Adams's wit, but it was a bit disappointing. Read more
Published on Nov 4 2001 by Kristopher Vincent

??? **A Terrible End to A Great Series**
The ending for this books was so bad that I vowed never to read another Douglas Adams book. Adams was obviously sick and tired of the series and used this book to kill it off with...
Read more
Published on Oct 17 2001 by David A. Lessnau

critiques de films

-1 **An insult to Douglas Adams' memory**
I agree entirely with "darkgenius'" comments. This movie is a travesty of the book and the TV series; a cutesy version totally lacking in the wit and satire of the original. Read more
Published 5 months ago by John W Beare

+1 **Don't Panic!**
If you haven't listened to the BBC radio-play, this isn't bad! Purists, no doubt, will dispute my verdict but the fact of the matter is THGTTG (see title) does have Douglas Adams'...
Read more
Published on Mar 13 2011 by Sid Matheson

+1 **On Blu-ray, even better**
I've seen this movie on TV and wanted to add it to my collection. I couldn't find it locally so when I saw it on amazon and on Blu-ray, I picked it up. Read more
Published on April 18 2009 by J. W. Little

-1 **An insult to Douglas Adams' memory**
The filmmaker's reverence for Adams' legacy? What kind of rubbish statement is that? As a loyal fan of Douglas Adams for more than a quarter of a century, I was appalled and...
Read more
Published on Aug 22 2006 by Daniel Jolley

Algorithme d'apprentissage

Classificateur

-1

Amazon reviews

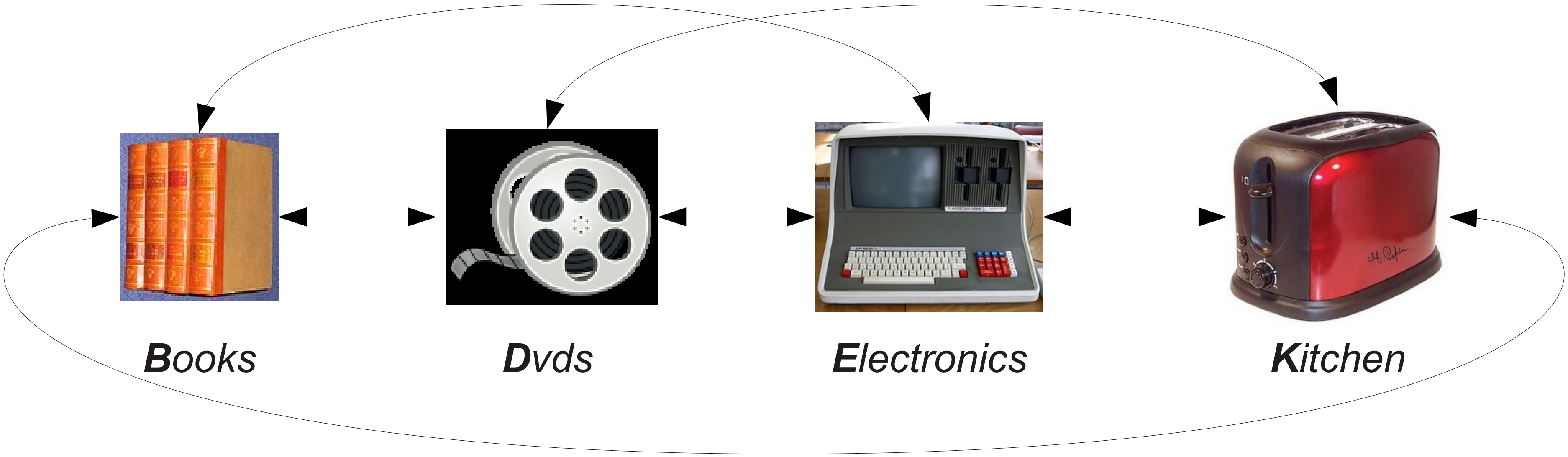


TABLE 2 – Taux d'erreurs sur *Amazon reviews*. B, D, E, K correspondent à *books*, *DVDs*, *electronics*, *kitchen*.

	B→D	B→E	B→K	D→B	D→E	D→K	E→B	E→D	E→K	K→B	K→D	K→E	Avg.
PBGD3^{CV}	*0.174*	0.275	0.236	*0.192*	0.256	0.211	0.268	0.245	*0.127*	0.255	0.244	0.235	0.226
SVM^{CV}	0.179	0.290	0.251	0.203	0.269	0.232	0.287	0.267	0.129	0.267	0.253	0.149	0.231
DASVM^{RCV}	0.193	*0.226*	*0.179*	0.202	*0.186*	0.183	0.305	*0.214*	0.149	0.259	*0.198*	0.157	*0.204*
CODA^{RCV}	0.181	0.232	0.215	0.217	0.214	*0.181*	0.275	0.239	0.134	*0.247*	0.238	0.153	0.210
PBDA^{RCV}	0.183	0.263	0.229	0.197	0.241	0.186	*0.232*	0.221	0.141	*0.247*	0.233	*0.129*	0.208

Fin.